West Park Primary

Respect Enjoy Aspire Achieve

Routes Through Calculations Policy

West Park Primary School
Pencil and paper procedures

Background to the policy

This policy contains the key pencil and paper procedures that will be taught within our school. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement in line with the expectations of the National curriculum.

Although the focus of the policy is on pencil and paper procedures it is important to recognise that the ability to calculate mentally lies at the heart of the national curriculum for mathematics. The mental methods will be taught systematically from Reception onwards and pupils will be given regular opportunities to develop the necessary skills. However mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it. In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Therefore written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

During their time at West Park school children will be encouraged to see mathematics as both a written and spoken language. Teachers will support and guide children through the following important stages:

- developing the use of pictures and a mixture of words and symbols to represent numerical activities;
- using standard symbols and conventions;
- use of jottings to aid a mental strategy;
- use of pencil and paper procedures;
- use of a calculator.

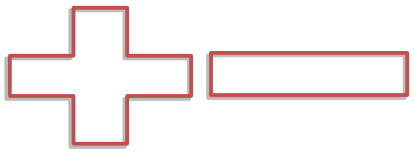
This policy concentrates on the introduction of standard symbols, the use of the empty number line as a jotting to aid mental calculation and on the introduction of pencil and paper procedures. It is important that children do not abandon jottings and mental methods once pencil and paper procedures are introduced. Therefore children will always be encouraged to look at a calculation/problem and then decide which is the best method to choose – pictures, mental calculation with or without jottings, structured recording or a calculator. Our aim is for children to be able to select an efficient method of their choice that is appropriate for a given task. They will do this by always asking themselves:

'Can I do this in my head?'
'Can I do this in my head using drawings or jottings?'
'Do I need to use a pencil and paper procedure?'
'Do I need a calculator?'

General progression

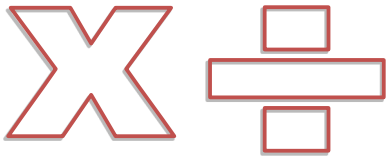
- Establish mental methods based on a good understanding of place value.
- Use of informal jottings to aid mental calculations.
- Develop use of empty number line to help mental imagery and aid recording.
- Use of partitioning and recombining to aid informal methods.
- Introduce expanded written method.
- Develop expanded written method into compact standard written method.

When are children ready for written calculations?



Addition and subtraction

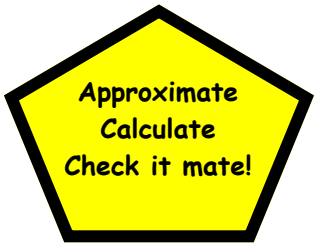
- Do they know addition and subtraction facts to 20
- Do they understand place value and can they partition numbers?
- Can they add three single digit numbers mentally
- Can they add and subtract any pair of two digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?



Multiplication and division

- Do they know the 2,3,4,5,and 10 times tables?
- Do they know the result of multiplying by 1 and 0
- Do they understand 0 as a place holder?
- Can they multiply 2 and three digit numbers by 10 and 100?
- Can they double and halve two digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?

It is important that any calculation is given a real life context or a problem solving approach to build children's understanding of the purpose of the problem. Also children will be encouraged to consider which is the best strategy to help them solve the problem.



When undertaking any calculations children will be asked to approximate, calculate and then to check it using an appropriate method. Children will be able to select the most efficient method for the numbers involved.

ADDITION

Children are taught to understand addition as combining sets and counting on. Practical apparatus will be used as well as pictures to develop children's understanding (models and images). Real life context will be provided to help children make links and to make learning purposeful.

3 + 2 =

3 people are on the bus. 2 more get on at the next stop. How many people are on the bus altogether

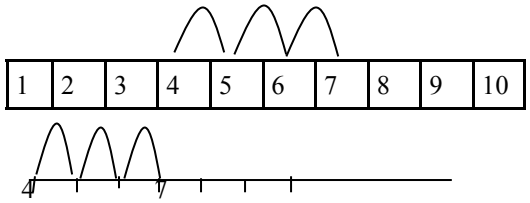
●●●● ●●●

Or

| | | | |+ | | |

4 + 3 =

Labelled number line moving to an unmarked number line



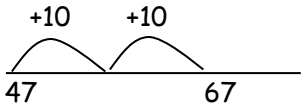
Stage 1

Children add numbers by counting on in ones

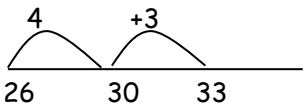
Children begin to use a marked number line to record the steps they have taken . Ensure children add on from the larger number . Once they are proficient with this they move on to using an unmarked number line

(bead strings can be used to show this)

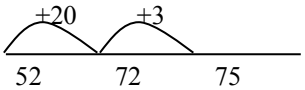
47+20



And **26 + 7**



Then 52 + 23



Stage 2

Add 2 digit numbers and tens

Add 2 digit numbers and units (*bonds of 10*)

Add pairs of two digit numbers moving to the expanded column when secure

Drawing an empty number line helps children to record the steps they have taken in a calculation

(use 100 squares, concrete equipment to build fluency in mental calculation)

<u>Partitioning both of the two digit numbers</u>

First 40 + 3 move to 50 + 6
20 + 5 30 + 8
60 + 8 = 68 80 + 14 = 94

Stage 3

This strategy develops children's understanding of place value. Children partition both the numbers into tens and units. They add the tens together and then the units and then go on to recombine the amounts to find the total. First only provide examples where chn do not need to cross the tens boundary . Once they are secure with this move onto crossing the tens boundary.

ADDITION

325 + 73

300 + 20 + 5
70 + 3
300 + 90 + 8 = 398

Stage 3 continued
Add numbers with up to 3 digits
When children are first taught written methods expanded sum is the first step in which the most significant (greater value) digits are added first. This way the children are more secure in the digit value eg 20 + 40 and not 2 + 4

Expanded Column Method -Least significant value
There are 324 boys and 245 girls in a school. How many children are there altogether?

324
+ 245
9
60
500
569

Stage 4
Add numbers with up to 3 digits
This step of the expanded method links to the compact method where children begin adding from the units column. It should be taught prior to the compact method as it gets children use to adding the least significant digits first.

Column method
2786 + 528=
2786 people visited the museum last year. The numbers increased by 568 this year. How many people altogether visited this year?

2 7 8 6
+ 5 2 8
3 3 1 4
1 1 1

Stage 5a
Add numbers with up to 4 digits
Once the children show a secure understanding of place value, they move to the efficient method of calculating—the compact column method. Children will use this method to add both whole and decimal values

12.786
+ 2.568
15.354
1 1 1

12.32
2.06
0.37
14.75
1

Stage 5b
Add numbers with more than 4 digit

23.456
2.600
50.380
1.030
77.466
1 1

234,567
3,607
51,709
39,712
329,595
1 1 1 2 2

Stage 5c
Add several numbers with increasing complexity
Adding numbers with different numbers of decimal places including money and measures
Children can put in zeros as place holders. They align whole and decimal digits accurately understanding tenths hundredths and thousandths values
They also add several numbers with more than 4 digits including whole numbers

SUBTRACTION

Children are taught to understand subtraction as taking away and finding the difference. Practical apparatus will be used as well as pictures to develop children's understanding (models and images). Real life context will be provided to help children make links and to make learning purposeful.

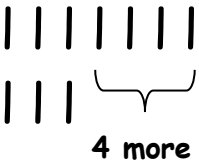
5-2=

I had five balloons. Two burst. How many did I have left?

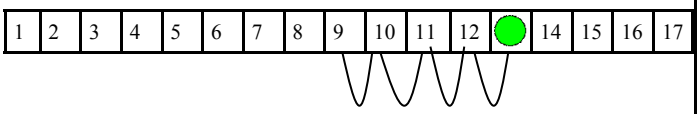
Adam has 5 marbles and Maya has 2. How many more does Adam have?

Find the difference

Ali has 7 felt tip pens and Tim has 3. How many more does Ali have?



Zak has 13 sweets. He eats four. How many are left?



Drawing a picture helps children to visualise the problem.
Later on the children will progress on to using dots or tally marks as a quicker way of drawing the problem

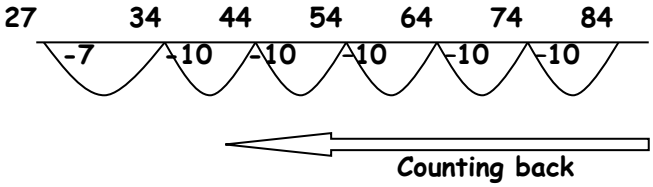
Stage 1
Subtract from numbers up to 20

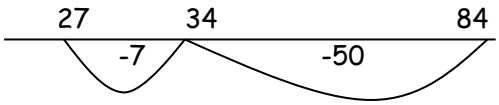
Children begin to use a marked number line to record the steps they have taken . Ensure children count back from the number they are subtracting from. Use a counter to support this. Once they are proficient with this they move on to using an unmarked number line
(Practical apparatus and bead strings)

SUBTRACTION

Take away –counting back

84-57





Stage 2

Subtract with 2-digit numbers

The number line is an example of a jotting that may help children keep track of their mental calculations.

They partition the number they are taking away into tens and ones. They takeaway the tens first and then the ones. As a starting point children may choose to count back 5 jumps of 10 under the number line to subtract 50. This is taking away.

When they are secure they could do this more efficiently by subtracting 50 and then the 7 Two jumps would be in the least amount of jumps.

(100 square, base 10 equipment)

Partitioned column subtraction

78 –46 =

Step 1

$$\begin{array}{rl} & 70 + 8 \\ - & 40 + 6 \\ \hline & 30 + 2 = 32 \end{array}$$

Step 2

$$\begin{array}{rl} & \overset{70}{\cancel{80}} + \overset{10}{1} \\ - & 30 + 6 \\ \hline & 40 + 5 = 45 \end{array}$$

Step 3

$$\begin{array}{rl} & \overset{100}{\cancel{200}} + 30 + \overset{1}{8} \\ & 100 + 60 + 7 \\ \hline & 0 + 70 + 1 = 71 \end{array}$$

Stage 3 Partitioning

Subtract with 2 and 3- digit numbers

Children partition both the numbers and set them out as shown. The starting point will be with no exchange

When introducing exchange support with base 10 equipment. Show the exchanged amount in place value. in step 2 the 1 ten is shown as a 10, which adds to the 1 to make 11. reinforcing place value.

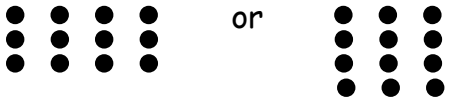
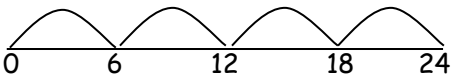
Once children are secure with exchange they can use this method to subtract any 2 and 3 digit number

SUBTRACTION

Step 1 2000 1000 50 1 ~~3000~~ + ~~400~~ + 60 + 3 2000 + 500 + 40 + 5 0 + 900 + 10 + 8 = 918 The above partitioning is not to be taught as a written method but only to be shown alongside dienes apparatus only 5 1 6 1 3 ~~6~~ ~~X~~ 2 2 4 9 3 1 1 7 9	**Stage 4** **Subtracting with up to 4 digit numbers** Provide opportunities for children to partition money and measures. Use practical equipment (dienes/base 10) for children to reinforce 'exchange'. Once children are confident with the partitioning and subtraction of numbers with up to 4 digits introduce them to the compact method. Those who are still not secure with number facts and place value and will need to remain on the partitioned column method
Compact method *6759- 3683.7 =* 6 1 8 1 6 ~~X~~ 5 ~~9~~ .0 - 3 6 8 3 . 7 L 3 0 7 5 . 3 L	Stage 5a **Subtract with at least 4-digit numbers** *including money ,measures an decimals* For children who are secure in their understanding of place value, this compact method will be appropriate. Encourage children to place a zero in the empty decimal place to fully grasp what they are subtracting from that column. Those who are still struggling will need to remain on the partitioned column method Always encourage chn to consider the best method for the numbers involved. This may include Mental, counting back, counting on or a written method.
0 1 4 1 4 ~~1~~ 2 6 9 ~~5~~ 4 - 5 2 8 2 6 4 0 7 4 1 2 8 2 13 1 ~~3~~ ~~4~~ . 6 7 5 km - 6 . 7 3 km 2 7 . 9 4 5 km	Stage 5b Subtract with increasingly large and complex numbers Children continue to use this strategy to subtract money and measure including decimals with different numbers of decimal places. Empty decimal places can be filled with zero in relevant columns. Continue to encourage children to select the most appropriate method for the numbers

MULTIPLICATION

Children are taught to understand multiplication as repeated addition. It can also described as an array. Practical apparatus will be used as well as pictures to develop children's understanding (models and images). Real life context will be provided to help children make links and to make learning purposeful.

Each child has two legs. How many legs do three children have?  2 + 2 + 2 = 6	<u>Stage 1</u> **Multiply with concrete objects, arrays and pictorial representations** Pictures are useful to help understanding. This begins to develops children's understanding of multiplication as repeated addition. Children needs practical solving opportunities to count in lots of 2s, 5s and 10s
4x3= A chew costs 4p. How much do 3 chews cost? **3 x 4** or **4 x 3**  **6x4=** There are 4 cats. Each cat has 6 kittens. How many kittens are there altogether?  0 6 12 18 24	<u>Stage 2a</u> **Multiply using arrays and repeated addition (using at least 2s, 5s and 10s)** Drawing an array (3 rows of 4 or 3 columns of 4) gives children an image of the answer. It also helps develop the understanding that 4x3 is the same as 3x4, giving the same answer. (commutative law) (practical equipment) <u>Stage 2b</u> Children could count on in equal steps, recording each jump on an empty number line. This shows 4 jumps of 6. (bead strings)

MULTIPLICATION

Grid method

34 x 6 =

There are 16 felt tips in a packet. How many are there in 6 packets?

X	7	
30	210	
8	56	
	266	

Stage 3

Multiply a two digit by a single digit

Introduce the grid method. Numbers are partitioned into their place value. (distributive law) in to 10s and 1s . The two answers are then added together to find the product.

253 x 7

X	7	
200	1400	
50	350	
3	21	
	1771	

Stage 4

Multiply a two and three digit by a single digit

Once children are confident with multiplying two and three digit numbers by a single digit using this method and have demonstrated a good understanding of carrying when they have done addition move them onto the compact method - short multiplication

Short multiplication

2 5 1
x 8
2 0 0 8
4

Short multiplication

3 . 5 1
x 6
2 1 . 0 6
3

Long Multiplication

1 2 5 1
x 23
3 7 5 3 (1251x3)
2 5 0 2 0 (1251 x 20)
2 8 7 7 3

Stage 5

Multiply up to 4 digits by 1 or 2 digits

For children who are secure in their understanding of place value, recording may progress to short multiplication. Use this method to **multiply more than 4 digit by a single digit** including money, measures and **decimals with up to 2 d.p**

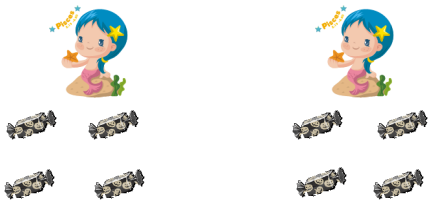
Long multiplication is used to multiply a number by a 2 digit

DIVISION

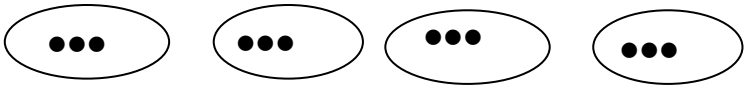
Children are taught to understand division as sharing and grouping. Practical apparatus will be used as well as pictures to develop children's understanding (models and images). Real life context will be provided to help children make links and to make learning purposeful.

8÷2=

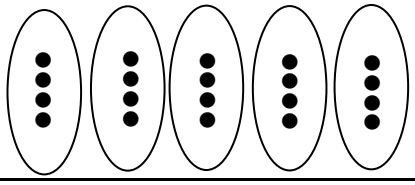
8 sweets are shared between 2 children. How many sweets do they get each?



How many groups of three can be made from 12 apples?

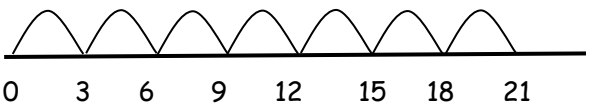


How many groups of 4 are in 20

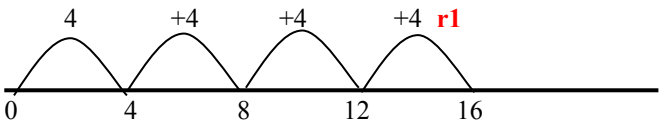


21÷3=

A chew bar costs 3p. How many can I buy with 21p?



17 ÷ 4 =



Stage 1

Group and share small quantities

Practical apparatus, pictures and arrays need to be used to support this understanding. Drawings often gives children a way into solving the problem. Children should be taught to understand the difference between sharing and grouping. Share these between 2 children. Group these by 2

Stage 2

Group and share using ÷ and =

Using the array children begin to understand 20 ÷ 4 = 5 and the same array horizontally represents 20 ÷ 5 = 4

Continue to develop understanding of sharing and grouping

Stage 3a

Divide a 2 digit number by a single digit (no remainder)

Counting on in equal jumps of the divisor on the number line . Draw jumps of 3 along a number line until you get to 21 counting from zero. This shows you need 7 jumps of 3 to reach 21. This reinforces the concept of grouping

Stage 3b

Children continue to develop understanding of unknown division facts by grouping with remainders using the number line. Chn should work towards doing the above mentally. This will support carrying when moving on to the short method (*Bead string, cubes)/straws*

DIVISION

84÷4=

I need 4 drawing pins to put up a picture. How many pictures can I put up with 84 pins?

Step 1

$$4\overline{)84} \quad 21$$

Step 2 With remainders

$$3\overline{)715} \quad 25$$

Stage 4

Divide a 2 digit number by a single digit (no remainder)

To begin ensure each digit in the number is the multiple of the divisor so there are no remainders

Move on to numbers with remainders

$$6\overline{)7^{1}6^{4}2} \quad 127$$

$$6\overline{)3^{3}5^{5}4} \quad 059$$

Stage 5

Divide up to 3 digit numbers (without remainders to begin)

Chn begin to divide 3 digit numbers without remainders to begin as in stage 3 and the move on to remainders

When working with a number where the answer to the first digit is a zero, chn can to begin with, place a zero to show its place and must always carry the digit over to the next digit as a remainder.

$$6\overline{)3^{3}8^{2}4\,8} \quad 0641\ r2$$

Stage 6

Dividing up to 4 digit numbers by a single digit (including remainders)

Continue to ensure all division has a real life context as in this stage chn will be expected to consider the remainder and how to best express it. i.e as a fraction, decimal or a rounded number depending on the context of the problem provided

$$6\overline{)3^{3}8^{2}4\,8.^{2}0^{2}0} \quad 0641.33$$

This example shows how to calculate a decimal remainder. Instead of expressing the remainder as a 2, a decimal point is added after the units and a decimal point is placed and the 2 remainder is carried on to the zero showing their was no decimal value in the original number. Keep dividing to the degree of accuracy required for the problem.

Stage 7

Divide at least a 4 digit number by both a single digit and 2 digit numbers including decimal numbers and quantities

Continue to use the above method but ensure the real life context of the problem provides chn with opportunities to consider whether to express the remainder as a fraction, decimal or a rounded number

DIVISION

884÷34=

Key facts

34) **8 8 4**

- 6 8 0 (x20 groups)

2 0 4

- 1 7 0 (x 5 groups)

3 4

- 3 4 x1 group)

0

x	34
1	34
2	68
4	136
10	340
5	170
20	680

Stage 7b

Divide at least a 4 digit number by both a single digit and 2 digit numbers including decimal numbers and quantities

For division by a 2 digit number introduce chn to long division by chunking method

Children record a key facts list to support them with the calculation. They can see what chunks are possible. Which makes their calculation more efficient

Add up the groups to get the answer and record the remainder where there is one